Bertrand competition

Consider a Bertrand oligopoly where two firms compete in prices in a market with linear demand. The market demand function is given by:

$$Q = a - bP, \tag{1}$$

where Q is the total quantity demanded in the market, P is the price, and a and b are positive constants. Both firms have different but constant marginal costs, denoted by c_1 and c_2. Find the equilibrium prices and profits for both firms.

Solution

Pricing Strategies

In a Bertrand oligopoly, firms choose prices simultaneously to maximize their profits. Let P_1 and P_2 be the prices set by firm 1 and firm 2, respectively. The demand for each firm depends on their relative prices:

If $P_1 < P_2$, firm 1 captures the entire market demand, $Q_1 = a - bP_1$, and $Q_2 = 0$. If $P_1 > P_2$, firm 2 captures the entire market demand, $Q_1 = 0$, and $Q_2 = a - bP_2$. If $P_1 = P_2$, both firms share the market demand equally, $Q_1 = Q_2 = \frac{1}{2}(a - bP_1)$.

Profit Functions

The profit function for each firm is given by revenue minus cost:

For firm 1:

$$\pi_1(Q_1) = P_1Q_1 - c_1Q_1 = (P_1 - c_1)Q_1, \tag{2}$$

For firm 2:

$$\pi_2(Q_2) = P_2Q_2 - c_2Q_2 = (P_2 - c_2)Q_2. \tag{3}$$

Equilibrium Prices

In a Bertrand oligopoly, firms have an incentive to undercut their competitor's price as long as their price is above their marginal cost. The equilibrium occurs when neither firm can increase its profit by changing its price. There are two possible scenarios:

If $c_1 < c_2$, firm 1 can undercut firm 2 by setting its price slightly below c_2, capturing the entire market demand. In this case, the equilibrium prices are $P_1^* = c_2$ and $P_2^* > c_2$. The profits are $\pi_1^* = (c_2 - c_1)(a - bc_2)$ and $\pi_2^* = 0$.

If $c_1 > c_2$, firm 2 can undercut firm 1 by setting its price slightly below c_1, capturing the entire market demand. In this case, the equilibrium prices are $P_1^* > c_1$ and $P_2^* = c_1$. The profits are $\pi_1^* = 0$ and $\pi_2^* = (c_1 - c_2)(a - bc_1)$.

If $c_1 = c_2$, both firms would set their prices equal to their common marginal cost, and they would share the market demand equally.